

ECON 4130  
2005 H

## Written Paper III (compulsary)

(To be submitted, Monday 7 November, at Ekspedisjonskontoret 12th floor ES.)

### Exercise 1

- a. Rice exercise 60 in chapter 2.
- b. Find  $E(Y)$  and  $\text{var}(Y)$  in **a.** [**Hint:** Consider the mgf of  $Z$ .]
- c. Find the median of  $Y$  in **a.**

### Exercise 2

**Rice exercise 38 in chapter 8.** The data for this exercise (in a STATA data file), can be downloaded from the course website (use the “save” option and not the “open” option on the menu that appears when you click the link to the data).

Answer first **a**, **b**, and **c**.

**Hint for c:** Maximum likelihood can be done in STATA by the ml-command, but somewhat involved. It is much easier to use Excel:

You don't need the whole data set in Excel, only the values of  $n$ ,  $\sum \ln x_i$ ,  $\sum x_i$ .

Choose two cells for the arguments,  $\alpha$ , and  $\lambda$ , to the log-likelihood, that you fill with suitable start values, e.g., the moment estimates from **b**. Choose also cells containing the three values above. Then define the log-likelihood function in a sixth cell (remember to start the function definition by an equality sign, =). Note also that the function,  $\ln \Gamma(\alpha)$  is implemented in Excel under the name GAMMALN. Click the cell where the log-likelihood function is, and use the solver- module (too be found under tools on the menu) to maximize the log likelihood.

[If you have never used Excel before, you are allowed to use the values 1,6 for  $\hat{\alpha}$ , and 2,6 for  $\hat{\lambda}$  (which are close to the mle's) for the rest of the exercise. State this in your answer.]

**Answer d.**

**Hint for d:** You need to use the “tway” graph command in STATA (see help twoway). You can combine several graphs in one by separating several graph commands by double vertical lines, `||`. For example, suppose the data are in column, `x`, and the values of a density, calculated for all values in `x`, are in a column, `g`. A graph that combines a histogram with the density plot, is for example made by the command

```
tway histogram x, bin(15) || line g x, sort
```

The option `bin(15)` says that the histogram shall contain 15 intervals. Choose the number of intervals yourself. The option, `sort`, to the line command sorts the data before plotting (try what happens without this option).

Alternatively you can use the menu “Overlaid twoway graphs” on the Graphics menu.

To calculate the gamma density, you can use the function, `gammaden(a,1,0,x)`, that calculates the  $\Gamma(a,1)$  density. (`gammaden(a,b,0,x)` does not seem to work properly for `b` different from 1). To calculate the  $\Gamma(a,b)$  density, you can instead use the following: If  $g(x)$  is the density for  $\Gamma(a,1)$ , then  $b \cdot g(bx)$  is the density for  $\Gamma(a,b)$ . Hence, the function, `b*gammaden(a,1,0,b*x)`, should do the trick.

**Skip e-f:** Instead of **e** and **f** in the book, calculate approximate standard errors for the mle estimators using the asymptotic theory described in “Lecture notes to Rice chapter 8”.

**Answer g.**